Revision — المرحلة الاعدادية :

N (nateral number) $= \{0, 1, 2, 3, ----\}$

$Z = Z^+ \cup \{0\} \cup Z^-$ (integer number) $= \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

Q (rational number) $= \{\frac{a}{b} : a, b \in Z, \ b \neq 0\}$

Q' (irrational number) $=$ not $\frac{a}{b}$ ex $\pi \simeq \frac{22}{7}$

R (real number) $= Q \cup Q' =]-\infty, \infty[= R^+ \cup \{0\} \cup R^-$

even number $\{0, 2, 4, 6, ---\}$

odd number $\{1, 3, 5, 7, ---\}$

Prime number $\{2, 3, 5, 7, 11, 13, 17, 19, ---\}$

- $(a \pm b)^2 = (\text{First})^2 \pm \text{First} \times \text{second} \times 2 + (\text{second})^2$

- $2x(-3y + 2x) = -6xy + 4x^2$

خد بالك

$+ \times + = +$

$- \times - = +$

$- \times + = -$

$+ \times - = -$

وزيهم ÷ بدل ×

- $(a+b)^3 = (a+b)^2 (a+b)$

$= (a^2 + 2ab + b^2)(a+b)$

$= a^3 + a^2 b + 2a^2 b + 2ab^2 + b^3$

$= a^3 + 3a^2 b + 3ab^2 + b^3$

- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + b^2$ not Factorize.
- $x^2 + 5x + 6 = 0$ Modde 5 : 3 او ع حسب نوع آلة
- $x - y = 3$, $x + y = 3$ Modde 51 ~ ~ ~
- $x + y + z = 5$, $y + z = 3$, $x - 2y = 7$ Modde 52 ~ "
- H.C.F : $21a^3b^2 - 7a^2b^2 - 35a^2b^3 =$

$7a^2b^2(a - 1 - 5b)$

بناخد من المتغير اقل Power ومن الارقام اللي يقبل القسمه عليهم

- The arithmetic mean of a set $= \dfrac{\text{Sum of these values}}{\text{Number of these values}}$
- The median ترتيب - ناخد اللى فى النصف ولو زوجى
  يبقى المرتبين اللى فى النصف
- The mode of set of values . القيمة الأكثر انتشاراً
- $P(A) = \dfrac{\text{The number of event}}{\text{number of sample space}} = \dfrac{n(A)}{n(S)}$
- $a^m \times a^n = a^{m+n}$ • $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- $a^m \div a^n = a^{m-n}$ اللى جوه اللى بره
- $(a^m)^n = a^{m \times n}$
- $a^{-m} = \dfrac{1}{a^m}$
- $(a \times b)^m = a^m \times b^m$
- $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
- $a^0 = 1 \Rightarrow$ شرط $a \neq 0$
- $a^m = a^n \therefore m = n \Rightarrow a \notin \{0, 1, -1\}$
- $a^m = b^m \Rightarrow a = b$ if m odd
  $a = \pm b$ if m even

ترتيب العمليات الحسابية ① اللى داخل القوس أول
② Power
③ $\div$ ، $\times$ قبل $+$ ، $-$ اللى يجى أولاً هو الأول

- $\sqrt{16} = 4$
- $-\sqrt{16} = -4$
- $\sqrt{-16}$ meaningless
- Two roots of 16 $\Rightarrow \pm 4$
- $\sqrt{a^2} = |a|$
- $(\sqrt{a})^2 = a$

- $2x-5=5 \Rightarrow 2x=10 \Rightarrow x=5 \quad \therefore S.S=\{5\}$
- $x>5 \quad]5,\infty[$ • $x \geq 5 \quad [5,\infty[$
- $x<5 \quad]-\infty,5[$ • $x \leq 5 \quad]-\infty,5]$

$-2<x<3 \Rightarrow x \in]-2,3[$

$-2 \leq x \leq 3 \Rightarrow x \in [-2,3]$

$-2<x \leq 3 \Rightarrow x \in]-2,3]$

$-2 \leq x<3 \Rightarrow x \in [-2,3[$

② $ax^2+bx+c=0 \quad S.S = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

a coff x^2, b coff x, c free Term

$L+M = \dfrac{-b}{a}$ sum of Two roots

$LM = \dfrac{c}{a}$ product of Two roots.

$x^2 -$ sum $x +$ product $= 0$ equation

$b^2-4ac \begin{cases} >0 & \text{Two roots real diff.} \\ =0 & \text{Two roots real equals} \\ <0 & \text{Two roots not real (complex).} \end{cases}$

$L^2+M^2 = (L+M)^2-2LM$

$L^3+M^3 = (L+M)((L+M)^2-3LM)$.

$\dfrac{1}{L}+\dfrac{1}{M} = \dfrac{L+M}{LM}$

• Domain of The Function:

① all The Polynomal Function domain is R

power $x \in N$ $\quad F(x)=3x^2+4x+7$

② Friction Domain $= R-\{$set Zero of Dominator$\}$

$F(x)=\dfrac{x}{x-2} \qquad R-\{2\}$

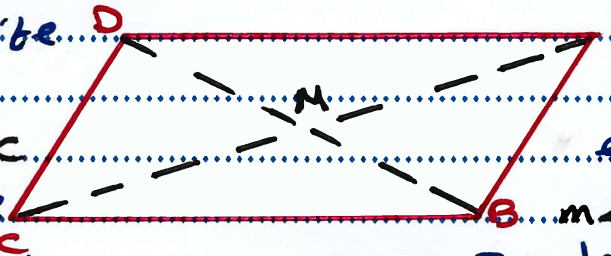
③ $\sqrt[\text{even}]{\bigcirc} \geq 0 \Rightarrow F(x)=\sqrt{x-3} \quad \therefore x-3 \geq 0$

$x \geq 3 \quad [3,\infty[$

④ $\sqrt[\text{odd}]{}$ Domain is $R \Rightarrow F(x)=\sqrt[3]{x+5}$

R

A Parallelogram



⊙ Each Two opposite sides are equals
$AB = DC$, $AD = BC$

⊙ Each Two opposite sides are Parallel
$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

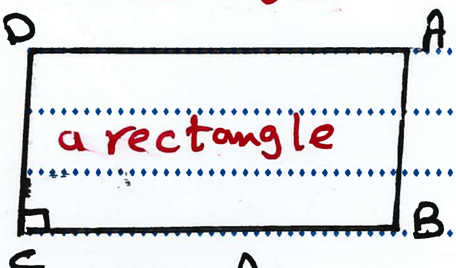
⊙ each Two opposite angles are equals $m\angle A = m\angle C$
$m\angle D = m\angle B$

⊙ The Two diagonals bisect each other $AM = MC$
$DM = MB$.

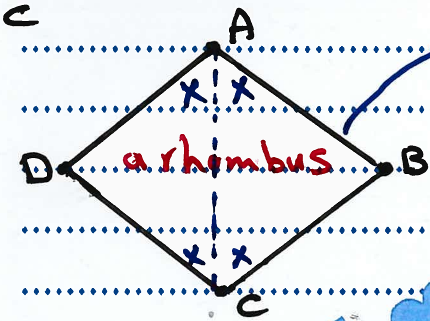
⊙ $m\angle A + m\angle B = 180°$, $m\angle B + m\angle C = 180°$,
$m\angle C + m\angle D = 180°$, $m\angle D + m\angle A = 180°$.

a square , a rhombus , a rectangle

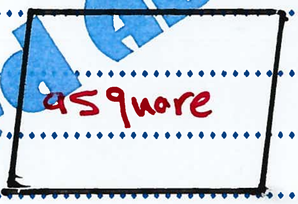
نفس الخواص دي هو موجود في
وكل واحد فيه حاجه زيادة



⊙ angles equals $90°$
⊙ Two diagonals are equals.

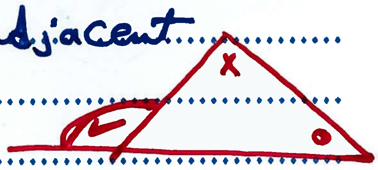


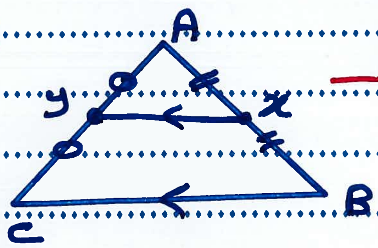
⊙ diagonals bisect angle of vertices.
⊙ all sides are equals
⊙ Two diagonals are perpendicular



⊙ كل الخواص فيه
ومفيش باي حاجه
بكم عن كل حاجه

⊙ The sum of The measures of The interior angles of a $\triangle = 180$
⊙ " " " " a $\square = 360$
⊙ The measure of The exterior angle of a triangle is equal to The sum of The measures of its non adjacent interior angles. $\checkmark = \circ + x$

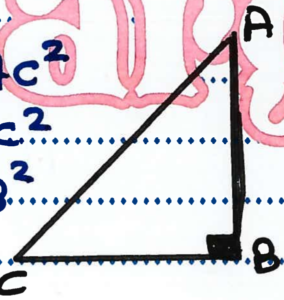




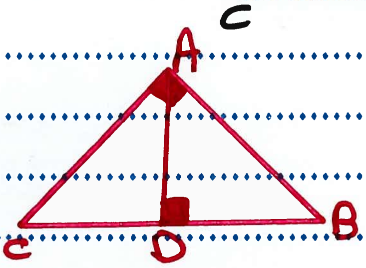
⊙ if x is mid Point $\overline{AB}$
if y is mid Point $\overline{AC}$
$\therefore \overline{xy} \parallel \overline{CB}$ and $xy = \frac{1}{2} CB$

The image of The Point (x, y)

- by reFlection in x-axis $(x, -y)$
- 〃 〃 〃 y-axis $(-x, y)$
- 〃 〃 〃 origin Point $(-x, -y)$
- by Translation (k, L) $(x+k, y+L)$
- by rotation $(0, 90°)$ $(-y, x)$
- by rotation $(0, 270°)$ $(y, -x)$
- by rotation $(0, \pm 180)$ $(-x, -y)$
- by rotation $(0, \pm 360)$ (x, y)



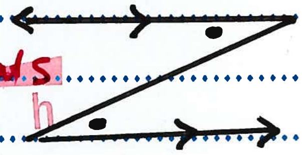
- $AB^2 + BC^2 = AC^2$
- $AB^2 = AC^2 - BC^2$
- $CB^2 = AC^2 - AB^2$



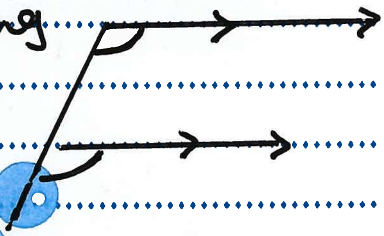
- $AB^2 = BD \times BC$
- $AC^2 = CD \times CB$
- $AD^2 = BD \times DC$
- $AD = \frac{AB \times AC}{BC}$

- Two Δ are Congruence.
  1. Two sides and The included angle (S-A-S)
  2. Two angles and one side (A.S.A)
  3. Three sides (S.S.S)
  4. Hypotenuse and one side (R.H.S)

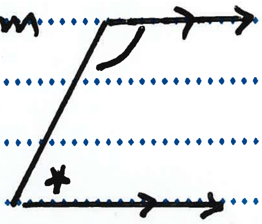
- alternat are equals Z



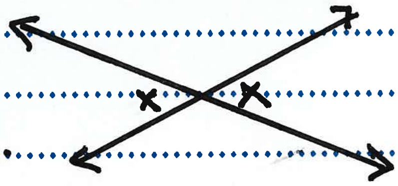
- crossPonding are equals F



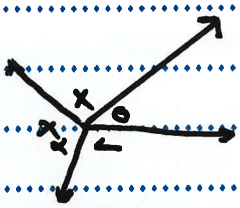
- interior sum = 180



- V.O.A are equals.

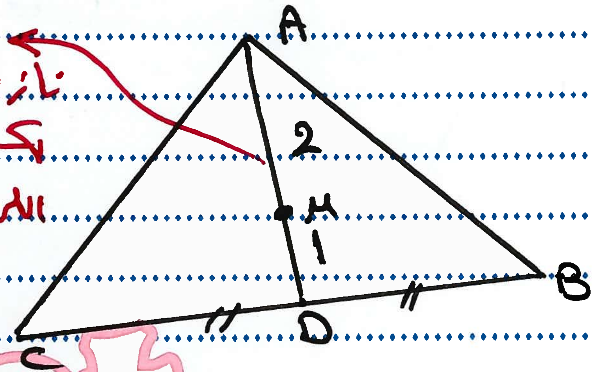


- Sum at Point = 360



- Sum measures of interior angles of any Polygon $= (n-2) \times 180$
- one angle in regular Polygon $= \frac{(n-2) \times 180}{n}$

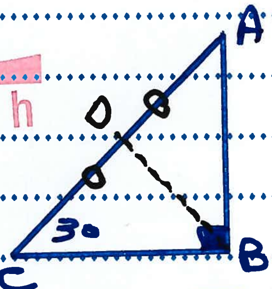
- Sum of measures of The Two Complementary angles is 90°
- Sum of measures of The Two Supplementary angles is 180°
- $\overline{AD}$ is median



bisect base يعمل vertiex ماذا:
bisect vericex و ⊥ عكسه مش
الا لو كان Δ issoseles او equilateral Δ

- M نقطة تلاقي المتوسطات
→ concurrent.

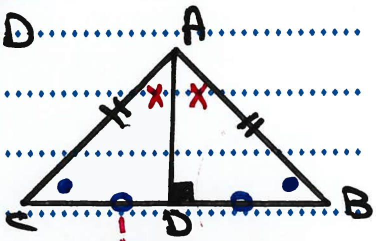
يتقسم بالنسبة 2 إلى 1 من جهة الرأس



- $AB = \frac{1}{2} AC$
$CB = \frac{\sqrt{3}}{2} AC$

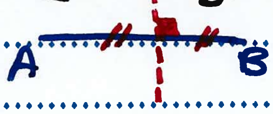
$BD = \frac{1}{2} AC$ → بـ مش ⊥

- The Sum of Two Sides in any Δ greater Than The Third Side.

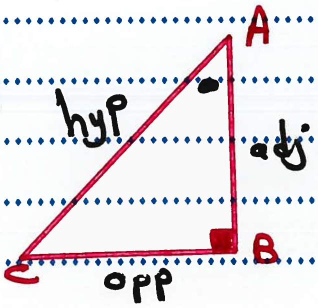


- if $AB = AC$
∴ if $\overline{AD} \perp \overline{BC}$ ∴ → $m\angle BAD = m\angle CAD$ → $BD = DC$
equilateral Δ ونفس الكلام في

- The axis of symmetry → bisect ⊥



- in any Δ The greatest Side opposite The greatest angle.
- in any Δ The Smallest Side opposite The Smallest angle.



- $\sin A = \frac{opp}{hyp} = \frac{CB}{AC}$
$\cos A = \frac{adj}{hyp} = \frac{AB}{AC}$
$\tan A = \frac{opp}{adj} = \frac{CB}{AB}$

- if $\sin A = \cos C$
∴ $A + C = 90^\circ$
- in The opp Figure
$\sin A = \cos B$
$\cos A = \sin B$.

- if $A(x_1, y_1)$, $B(x_2, y_2)$
  - $\to AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
  - $\to$ mid Point of AB $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
  - $\to$ slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

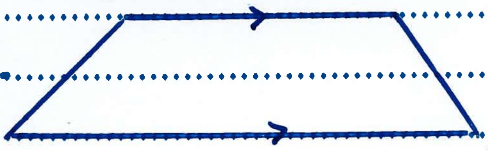
- Slope
  - $\to$ given Two Points on the line as: $A(x_1, y_1)$, $B(x_2, y_2)$ $\quad \dfrac{y_2 - y_1}{x_2 - x_1}$
  - $\to m = \tan\theta \to$ angle with the direction x-axis.
  - $\to y = bx + c \to$ slope b بشرط y لوحده
  - $\to ax + by + c = 0 \to$ slope $\dfrac{-b}{a}$ لو y ، x في نفس الجهة

- $L_1 \parallel L_2 \quad \therefore m_1 = m_2$
  $L_1 \perp L_2 \quad m_1 \times m_2 = -1$

- Prove A, B, C Three Points in one Coliner.
  ① The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{BC}$
  or $AB + BC = AC$ (where AC is The greatest Length).

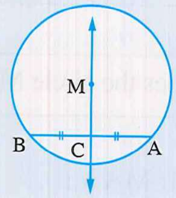
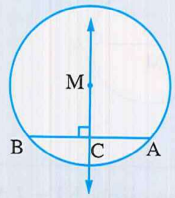
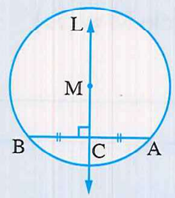
- if AC greatest side Then $AC^2 = AB^2 + BC^2$ Δ right
  $AC^2 > AB^2 + BC^2$ Δ obtuse
  $AC^2 < AB^2 + BC^2$ Δ acute.

- Trepezum:
  Two sides Parrel and not equals.
  issoseles Trapezum (لو عندي)

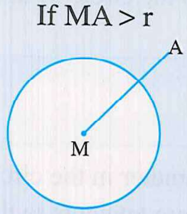
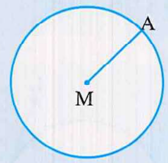
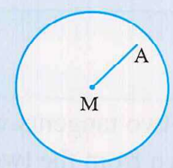


angle base are equals يبقى

Final revision on geometry

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.		If $\overline{AB}$ is a chord of the circle M and C is the midpoint of $\overline{AB}$, then : $\overleftrightarrow{MC} \perp \overline{AB}$
The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.		If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then : C is the midpoint of $\overline{AB}$
The perpendicular bisector to any chord of a circle passes through the centre of the circle.		If $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$ and the straight line $L \perp \overline{AB}$ from the point C , then $M \in$ the straight line L

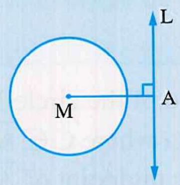
Position of a point with respect to a given circle

A is outside the circle M	A is on the circle M	A is inside the circle M
If $MA > r$	If $MA = r$	If $MA < r$
		

Position of a straight line L with respect to a circle M which at distance MA from its centre

L lies outside the circle M

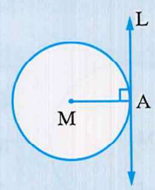
If $MA > r$



- $L \cap$ the circle $M = \varnothing$
- $L \cap$ the surface of the circle $M = \varnothing$

L touches the circle M

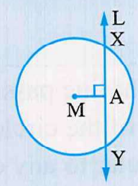
If $MA = r$



- $L \cap$ the circle $M = \{A\}$
- $L \cap$ the surface of the circle $M = \{A\}$

L is a secant to the circle M

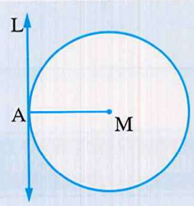
If $MA < r$



- $L \cap$ the circle $M = \{X , Y\}$
- $L \cap$ the surface of the circle $M = \overline{XY}$

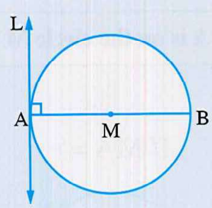
$\overline{XY}$ is called the chord of intersection

The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



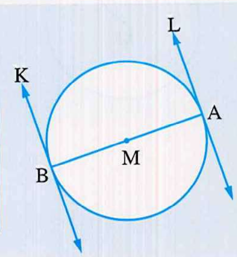
If L is a tangent to the circle M at the point A , then $\overline{MA} \perp L$

The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



If $\overline{AB}$ is a diameter of the circle M , $L \perp \overline{AB}$ at the point A , then L is a tangent to the circle M at the point A

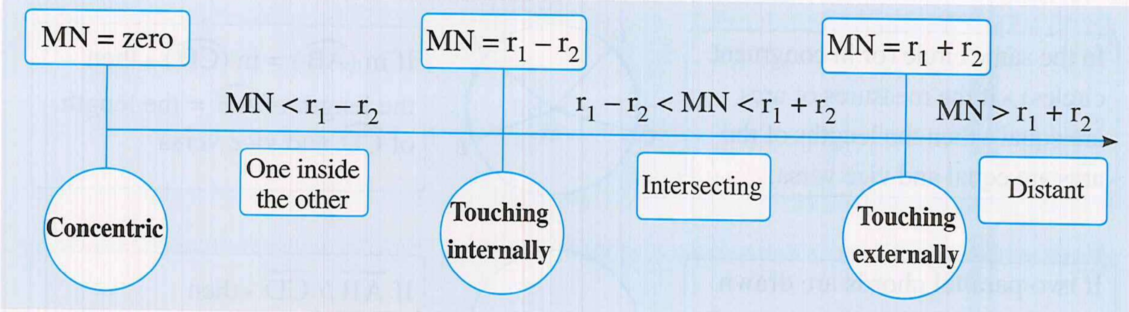
The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.



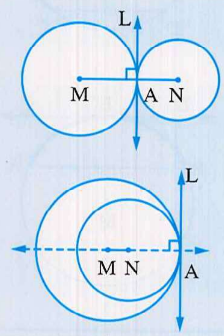
If $\overline{AB}$ is a diameter in the circle M , L and K are tangents to the circle M at A , B , then L // K

Position of the circle M with respect to the circle N

To determine the position of the circle M (with radius length r_1) with respect to the circle N (with radius length r_2) , find $r_1 - r_2$, $r_1 + r_2$, then use the following diagram to determine the position (where $r_1 > r_2$)

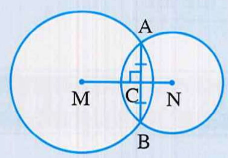


The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.



If the two circles M and N are touching at A , L is a common tangent to them at A , then $\overleftrightarrow{MN} \perp L$

The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.



If M and N are intersecting circles at A and B , then $\overleftrightarrow{MN} \perp \overline{AB}$, $AC = BC$

($\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$)

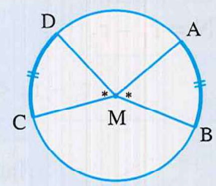
Remarks on identifying the circle

- It is possible to draw an infinite number of circles passing through a given point.
- There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$
- The smallest circle passes through the two points A , B is the circle in which $\overline{AB}$ is a diameter in it and its centre is the midpoint of $\overline{AB}$ and length of its radius = $\frac{1}{2}$ AB
- It is impossible to draw a circle passing through three collinear points.

- There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

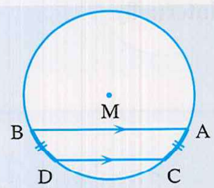
Equality of arcs in measure and length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.



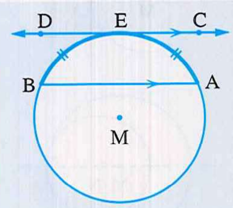
If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$, then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$ and vice versa

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.



If $\overline{AB} \,//\, \overline{CD}$, then $m(\overset{\frown}{AC}) = m(\overset{\frown}{BD})$

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.



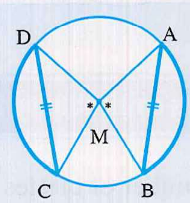
If $\overleftrightarrow{CD} \,//\, \overline{AB}$, then $m(\overset{\frown}{EA}) = m(\overset{\frown}{EB})$

Notice that

$$\frac{\text{The measure of the arc}}{360^\circ} = \frac{\text{The length of the arc}}{2\pi r}$$

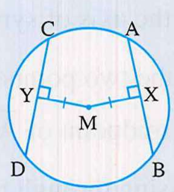
Equality of two chords in length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length and vice versa.



If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$, then AB = CD and vice versa

In the same circle (or in congruent circles) , if chords of a circle are equal in length , then they are equidistant from the centre and vice versa.

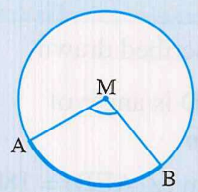


If AB = CD , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, then MX = MY and vice versa

Central angle , inscribed angle and angle of tangency and relation between them

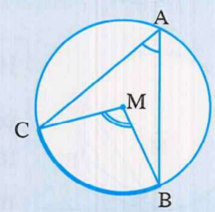
The measure of central angle

Equals the measure of subtended arc



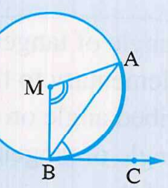
$m(\angle M) = m(\overset{\frown}{AB})$

Equals twice the measure of the inscribed angle subtended by the same arc



$m(\angle M) = 2\,m(\angle A)$

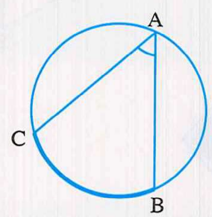
Equals twice the measure of the angle of tangency subtended by the same arc



$m(\angle M) = 2\,m(\angle ABC)$

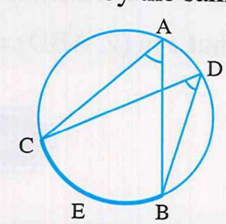
The measure of the inscribed angle

Equals half the measure of the subtended arc



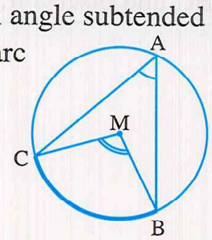
$m(\angle A) = \frac{1}{2}\,m(\overset{\frown}{BC})$

Equals the measure of the inscribed angle subtended by the same arc



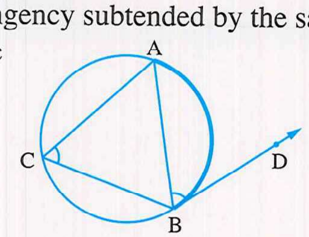
$m(\angle A) = m(\angle D)$

Equals half the measure of the central angle subtended by the same arc



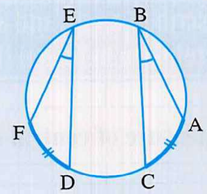
$m(\angle A) = \frac{1}{2}\,m(\angle M)$

Equals the measure of the angle of tangency subtended by the same arc



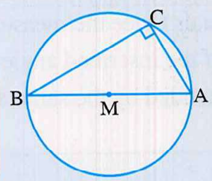
$m(\angle C) = m(\angle ABD)$

In the same circle (or in any number of circles), the measures of the inscribed angles subtended by arcs of equal measures are equal and vice versa.



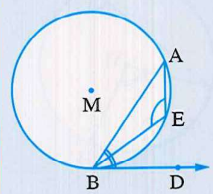
If $m(\overset{\frown}{AC}) = m(\overset{\frown}{DF})$, then $m(\angle B) = m(\angle E)$ and vice versa

The inscribed angle in a semicircle is a right angle



If $\overline{AB}$ is a diameter, then $m(\angle C) = 90°$

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

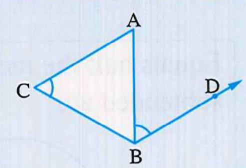


If $\angle AEB$ is inscribed drawn on $\overline{AB}$, $\angle ABD$ is angle of tangences, then $m(\angle ABD) + m(\angle AEB) = 180°$

Notice that

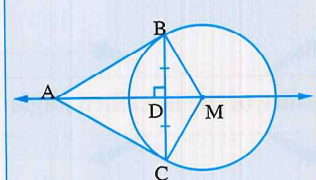
To prove that $\overleftrightarrow{BD}$ is a tangent to the circumcircle of ΔABC

Prove that : $m(\angle ABD) = m(\angle ACB)$



Relation between tangents of the circle

The two tangent-segments drawn to a circle from a point outside it are equal in length



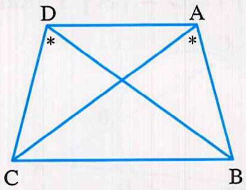
If $\overline{AB}$ and $\overline{AC}$ are tangent segments to the circle M, then

- $AB = AC$
- $\overleftrightarrow{AM}$ bisects $\angle BAC$ and $\angle BMC$
- $\overleftrightarrow{AM} \perp \overline{BC}$ and bisects it.

Cyclic quadrilateral

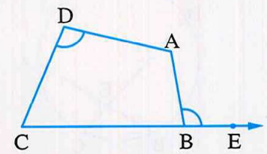
When is the quadrilateral cyclic ?

If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.



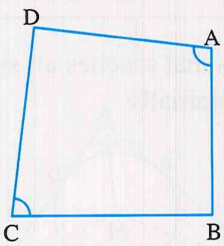
If m (∠ BAC) = m (∠ BDC)

If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.



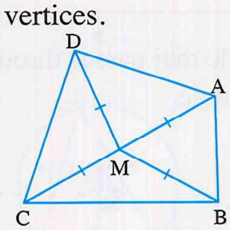
If m (∠ ABE) = m (∠ D)

If there are two opposite supplementary angles



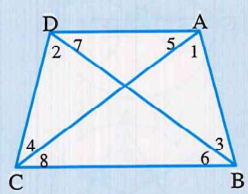
If m (∠ A) + m (∠ C) = 180°

If there is a point in the plane of the figure such that it is equidistant from its vertices.

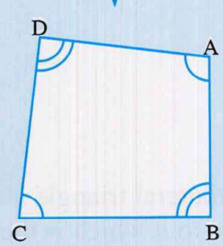


If MA = MB = MC = MD

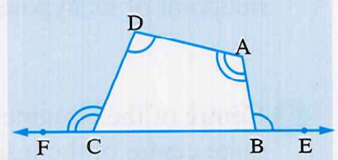
Properties of cyclic quadrilateral



- m (∠ 1) = m (∠ 2)
- m (∠ 3) = m (∠ 4)
- m (∠ 5) = m (∠ 6)
- m (∠ 7) = m (∠ 8)



- m (∠ A) + m (∠ C) = 180°
- m (∠ B) + m (∠ D) = 180°

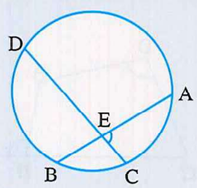


- m (∠ ABE) = m (∠ D)
- m (∠ DCF) = m (∠ A)

Well known problems

Well known problem (1)

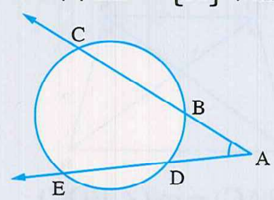
If $\overline{AB}$, $\overline{CD}$ are two chords in a circle intersecting at the point E , then :



$m(\angle AEC) = \frac{1}{2}[m(\widehat{AC}) + m(\widehat{BD})]$

Well known problem (2)

If $\overline{CB}$ and $\overline{ED}$ are two chords in a circle , where $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, then :

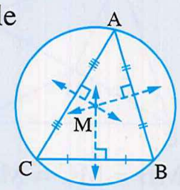


$m(\angle A) = \frac{1}{2}[m(\widehat{CE}) - m(\widehat{BD})]$

Circumcircle and inscribed a circle of the triangle

The circumcircle of the triangle

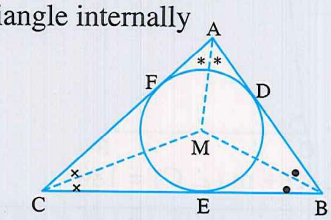
Is the circle that passes through vertices of the triangle



and its centre is the point of intersection of the perpendicular bisectors of its sides

The inscribed circle of the triangle

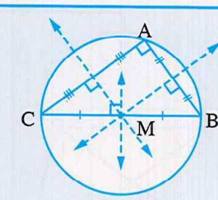
Is the circle is that touches all sides of the triangle internally



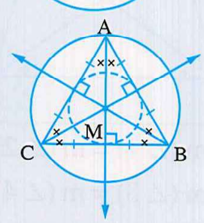
and its centre is the intersection point of the bisectors of its interior angles

Notice that

1. Centre of the circumcircle of the right-angled triangle is the midpoint of its hypotenuse.



2. Centre of the circumcircle of equilateral triangle is the same centre of the inscribed circle to it which is the point of intersection of axes of its sides and the point of intersection of its median and the point of intersection of the bisectors of its interior angles and also point of intersection of its altitudes.



3. It's possible to draw a circumcircle to a rectangle , a square and an isosceles trapezium while it's impossible to draw a circumcircle to a parallelogram , a rhombus and not isosceles trapezium.