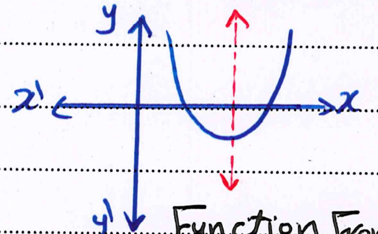
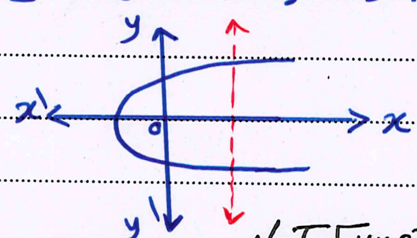
ملخص منهج البحتة

⇨ **Real Function:** The Function $f: x \longrightarrow y$ is Called a real Function if each of The domain (x) and The Co-Domain (y) and The image is Range.

⇨



Function From $x \longrightarrow y$



NoT Function From $x \longrightarrow y$

⇨ **Domain of Function**

خلي بالك ان Polynomial يعني
$\{0,1,2,3,\dots\} = \mathbb{N} \ni$ Power x

① all The Polynomial Function The Domain is R

② Domain of rational Function is R – set of zero of The denominator.

③ If $F(x) = \sqrt[n]{F(x)}$ where $n \in Z^+$, $n > 1$, $h(x)$ is Polynomial

(i) When n is odd number Then The Domain is R.

(ii) When n is even Then Domain $F(x) \geq 0$.

④ $\log_a^x$ $\quad x \in R^+$, $a \in R^+ - \{1\}$.

خلي بالك ان ش
بحث ١٠٠٪ Function

(i) if $F(x)$ is First degree : $F(x) = ax + b$

sine different a $\quad$ $-\frac{b}{a}$ $\quad$ sine same a

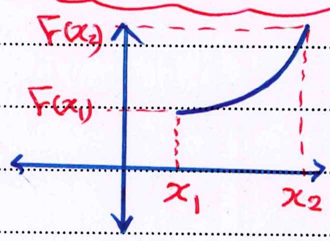
(ii) If $F(x)$ is second degree. Two roots L, M (real) : $F(x) = ax^2 + bx + c$

same a $\quad L \quad$ different a $\quad M \quad$ same a

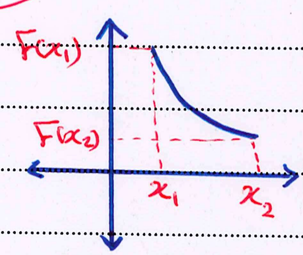
* one roots L $\quad$ same a $\quad L \quad$ same a

* not roots real $\quad$ all The sine of a

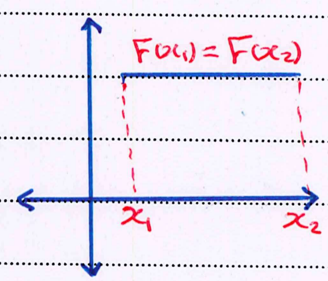
⇨ **monotony of Function**



increasing $]x_1, x_2[$



decreasing $]x_1, x_2[$



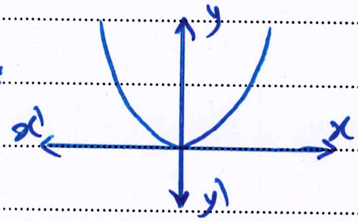
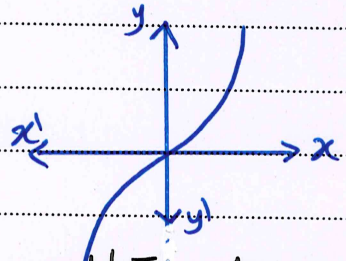
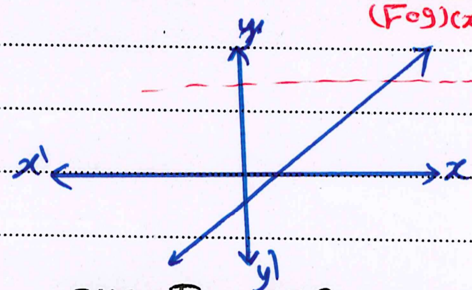
Constant $]x_1, x_2[$

* خلي بالك علشان نجيب $(F_1 \pm_{\times} F_2)(x)$ لازم نجيب $D_1 \cap D_2$

اما لو عاوز $(F_1 \div F_2)(x)$ لازم نجيب $D_1 \cap D_2 - Z(F_2)$

* $F\circ g(x) = F(g(x))$. To determine the domain of Function $(F\circ g)$ do as following
(1) Find: D_1 = Domain of g (2) Find: D_2 = Values of x that make $g(x)$ in domain of F (3) Find: $D_1 \cap D_2$ which is the domain of $(F\circ g)$

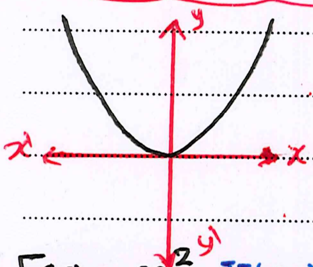
ملحوظة $(F\circ g)(x) \neq (g\circ F)(x)$

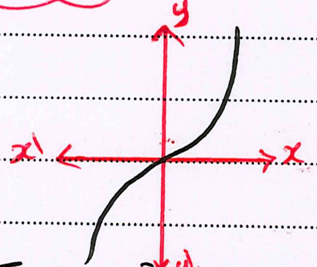
*   

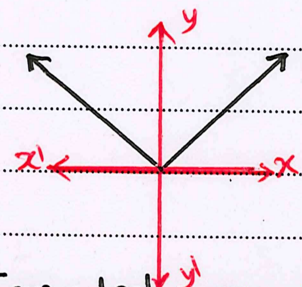
even Function	odd Function	one-To-one
$F(-x) = F(x)$	$F(-x) = -F(x)$	if $F(a)=F(b)$ $\therefore a=b$
Symmetric about y-axis	Symmetric about O origin Point	horizental line cut one Point

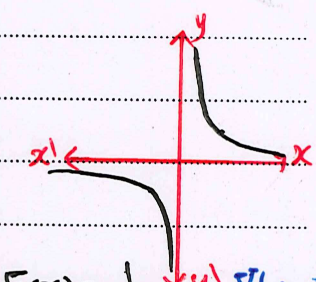
* if F_1, F_2 is an even, g_1, g_2 is an odd Function Then
(i) $F_1 \pm F_2$ is even (ii) $g_1 \pm g_2$ is odd (iii) $F_1 \pm g_1$ is neither even nor odd
(V) each of $F_1 \times F_2$ and $\frac{F_1}{F_2}$ is even (iV) each of $g_1 \times g_2$ and $\frac{g_1}{g_2}$ is even
(Vi) each of $F_1 \times g_1$ and F_1/g_1 is odd.

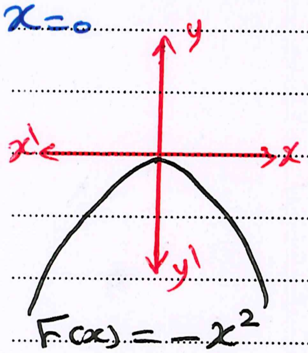
* Graphical Function :-

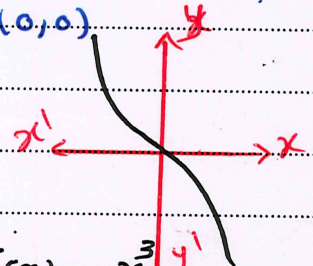

$F(x) = x^2$ $V(0,0)$
have axis of symmetric $x=0$

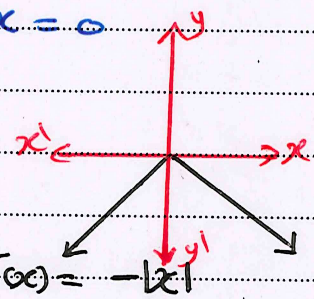

$F(x) = x^3$ $V(0,0)$
have Point of symmetric $(0,0)$

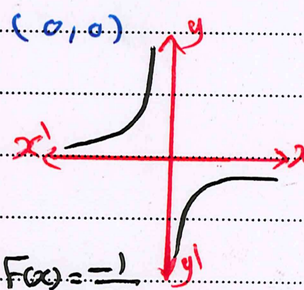

$F(x) = |x|$ $V(0,0)$
have axis of symmetric $x=0$


$F(x) = \frac{1}{x}$ $V(0,0)$
have Point of symmetric $(0,0)$


$F(x) = -x^2$


$F(x) = -x^3$


$F(x) = -|x|$


$F(x) = \frac{-1}{x}$

equation $F(x) = ax^2 + bx + c$ $\therefore V = \left(\frac{-b}{2a}, F\left(-\frac{b}{2a}\right)\right)$ خلي بالك

* The image of The curve $y = F(x)$ by reflection
  - in y-axis $\longrightarrow$ $y = F(-x)$
  - in x-axis $\longrightarrow$ $y = -F(x)$
  - in origin Point $\longrightarrow$ $y = -F(-x)$

* Solving absolute value :-

(i) $|ax+b| = \begin{cases} ax+b, & x \geq -\frac{b}{a} \\ -ax-b, & x < -\frac{b}{a} \end{cases}$

(ii) $|a| \geq 0$ (iii) $|ab| = |a||b|$

(v) $|a+b| \leq |a| + |b|$ (iv) $|a| = |-a|$

(vi) $|a-x| = |x-a|$

* $|x| = c,\ c > 0$ $\therefore x = \pm c$ * $|a| = |b|$ $\therefore a = \pm b$

* $(|a|)^2 = a^2$ * $\sqrt{a^2} = |a|$ * IF $|x| < a$ $\therefore -a < x < a$

* $|x| \leq a$ $\therefore -a \leq x \leq a$ * $|x| > a$ $\therefore x > a$ or $x < -a$

$x \in R - [-a, a]$

* $|x| \geq a$ $\therefore x \geq a$ or $x \leq -a$ $\therefore x \in R -]-a, a[$

* exponential :-

① $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ ② $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

③ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ ④ $\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}$.

⑤ $a^0 = 1$ $a \neq 0$ ⑥ $a^{-n} = \frac{1}{a^n}$ ⑦ $a^m \times a^n = a^{m+n}$ ⑧ $a^m \div a^n = a^{m-n}$

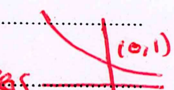
⑨ $(a^m)^n = a^{mn}$ ⑩ $(ab)^n = a^n b^n$ ⑪ $(\frac{a}{b})^{-n} = (\frac{b}{a})^n = \frac{b^n}{a^n}$

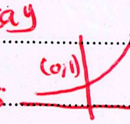
⑫ $a^m = a^n$ $\therefore n = m$ $a \neq \{0, 1, -1\}$

⑬ $a^m = b^m$ $\therefore a = b$ if n odd

$a = \pm b$ if n even.

⑭ $F(x) = a^x$ $a \in R^+ - \{1\}$ → $0 < a < 1$ decreas decay / → $a > 1$ increas growth





→ Domain is R → range R^+

→ one-To-one → $a^x \to \infty$ when $x \to \infty$ where $a > 0$

→ $a^x \to 0$ when $x \to \infty$ where $0 < a < 1$

$y = a^x$ Then $F(-x) = a^{-x} = (\frac{1}{a})^x$ is The image of curve by reflex y-axis

→ Exponential grawTh: $F(t) = a(1+r)^t$ } $A = P(1+\frac{r}{n})^{nt}$ لونيه تركيبيه

→ Exponential decay: $F(t) = a(1-r)^t$ where a is initial value, r is perecatage (n is Time

The inverse Function :-

① $F \circ g(x) = x$, $g \circ F(x) = x$

② domain f = range F^{-1}, range F = domain F^{-1}

③ one-To-one ④ Symmtic about $y = x$ خلى كل y x و كل x y

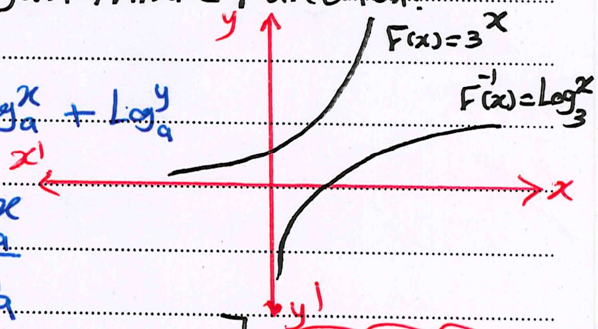
Logarithmic Function :

$y = \log_a x \iff x = a^y$ where $x \in R^+$

$a \in R^+ - \{1\}$, $y \in R$

* exponential Function is inverse of Logarithmic Function.

* Some properties of Logarithms:



① $\log_a a = 1$ ② $\log_a 1 = 0$ ③ $\log_a xy = \log_a x + \log_a y$

④ $\log_a \frac{x}{y} = \log_a x - \log_a y$

⑤ $\log_a x^n = n \log_a x$ ⑥ $\log_y x = \dfrac{\log_a x}{\log_a y}$

⑦ $\log_y x = \dfrac{1}{\log_x y}$ [if $x \in R^*$ and m is an even number $\neq 0$, $a \in R^+ - \{1\}$ Then $\log_a x^m = m \log_a |x|$]

(خلي بالك قوي)

⑧ $\log 10 = 1$ Ex: $\log_5 x^4 = 4 \log_5 |x|$ مهم جدا

⑨ If $\log_a x = \log_a y \quad \therefore x = y$

Second: Calculus :-

① The unspecified quantities are seven: $\frac{0}{0}$, $\infty - \infty$, $\frac{\infty}{\infty}$, $\infty \times 0$, 0^0, ∞^0, 1^∞

② $\frac{5}{0}$ is undefined quantity.

③ If $F(a^+) = F(a^-) = L \quad \therefore \lim\limits_{x \to a} F(x) = L$

اول خطوه في limit التعويض المباشر وبعد كده الطرح لو طلعت حاجه من 7 تحاول تشيل (CoFF zero)

Find The limit:

(i) if $F(x)$ is Polynomal Function in x Then $\lim\limits_{x \to a} F(x) = F(a)$

(ii) if $F(x) = k$ is Constant $\therefore \lim\limits_{x \to a} k = k$

(iii) if $\lim\limits_{x \to a} F(x) = L$, $\lim\limits_{x \to a} g(x) = m \quad \therefore$ ① $\lim\limits_{x \to a} (F(x) \pm g(x)) = L \pm m$

② $\lim\limits_{x \to a} F(x) \times g(x) = L \times m = \lim\limits_{x \to a} F(x) \times \lim\limits_{x \to a} g(x)$

③ $\lim\limits_{x \to a} k F(x) = k \lim\limits_{x \to a} F(x) = k F(a)$

④ $\lim\limits_{x \to a} F(x) \div g(x) = \dfrac{\lim\limits_{x \to a} F(x)}{\lim\limits_{x \to a} g(x)} = \dfrac{F(a)}{g(a)} = \dfrac{L}{m}$ بشرط $g(a) \neq 0$

⑤ $\lim\limits_{x \to a} (F(x))^n = \left(\lim\limits_{x \to a} F(x)\right)^n = L^n$, $n \in Z^+$

⑥ $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x^m - a^m} = \dfrac{n}{m} a^{n-m}$ where $n, m \neq 0$

⑦ $\lim\limits_{x\to\infty} \dfrac{k}{x} = 0$ ⑧ $\lim\limits_{x\to\infty} \dfrac{a}{x^n} = \text{Zero}, n \in R^+$ ⑨ $\lim\limits_{x\to\infty} x^n = \infty$

في حالة ∞ بنقسم على اعلى dominator في power

لو اعلى power فوق = اعلى power تحت خد coff ÷ coff

لو " " " > " " " يبقى $\pm\infty$

لو " " " < " " " يبقى zero

علي

⑨ $\lim\limits_{x\to 0} \dfrac{\sin ax}{bx} = a/b$ ⑩ $\lim\limits_{x\to 0} \dfrac{\tan ax}{bx} = \dfrac{a}{b}$

⑪ $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x} = 0$

تذكر أن

- $\sin^2 x + \cos^2 x = 1$ ①
- $1 + \tan^2 x = \sec^2 x$ ②
- $1 + \cot^2 x = \csc^2 x$ ③

خلي بالك: if x is the measure of an angle in the degree measure, Then $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = \dfrac{\pi}{180}$, $\lim\limits_{x\to 0} \dfrac{\tan x}{x} = \dfrac{\pi}{180}$

⇐ ج المائل ج $\dfrac{\pi}{2}, \dfrac{3}{2}\pi$ change, $\pi, 2\pi$ not change

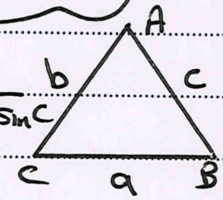
⑫ if $\lim\limits_{x\to a} F(x) = F(a)$ ∴ $F(x)$ is Continuity at $x = a$

مهم: all The Function Continuity in The Domain.

خلي بالك $F(x) = \tan x$ متصلة Continuity على $x \in R - \{\frac{\pi}{2} + n\pi, n \in Z\}$

Third: Trigonometry ∴

① $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2r = \dfrac{P\,of\,\triangle ABC}{\sin A + \sin B + \sin C}$



② $a^2 = b^2 + c^2 - 2bc\cos A$

③ $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

عدد حلول المثلث عن طريقة قانون cos لو التحليل طلع عددين موجبين يبقى عدد الحلول ② لو التحليل عدد موجب واخر سالب حل واحد لو التحليل طلع عددين سالبين او مركبين لا يوجد حلول.

The equation	General s.s
$\sin x = 0$	$x = \pi n$
$\sin x = 1$	$x = \frac{\pi}{2} + 2\pi n$
$\sin x = -1$	$x = \frac{3\pi}{2} + 2\pi n$
$\cos x = 0$	$x = \frac{\pi}{2} + \pi n$
$\cos x = 1$	$x = 2\pi n$
$\cos x = -1$	$x = \pi + 2\pi n$

خلي بالك cyclic

Two opp angle = 180

Two angle in same direction and same base equale

PURE :: ملخص المنهج

First : Algebra

(1) For $n \geq 1$: ⓐ $T_{n+1} - T_n > 0$ ∴ sequenc increasing

ⓑ $T_{n+1} - T_n < 0$ ∴ sequenc decreasing.

ⓒ $T_{n+1} - T_n = 0$ ∴ sequenc is constat

ⓓ If the sign of each element of a sequence differs from the sign of its next term, then the sequence is called an alternating sequence

(2) ⓐ $\sum_{r=1}^{n} c = cn$ ⓑ $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ ⓒ $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

ⓓ $\sum_{r=1}^{n} cT_r = c\sum_{r=1}^{n} T_r$ ⓔ $\sum_{r=1}^{n} T_r \pm E_r = \sum_{r=1}^{n} T_r \pm \sum_{r=1}^{n} E_r$

(3) Arithmetic sequence : ⊛ $T_n = a + (n-1)d$

First term → a ; n order of term ; d → $T_{n+1} - T_n$ → Constant

Last term ⊛ $L = a + (n-1)d$

⊛ if x, y, z are A.S ∴ $y = \frac{x+z}{2}$

⊛ $S_n = \frac{n}{2}(2a + (n-1)d)$ ⊛ $S_n = \frac{n}{2}(a + L)$

if T_x, T_y are two terms of an A.S خلي بالك السر يع
where $x \neq y$ then $d = \frac{T_y - T_x}{y - x}$

$a-d, \ a, \ a+d$ لو هنفرض Three Term in A.S

$a-3d, \ a-d, \ a+d, \ a+3d$ لو هنفرض Four Term

⊛ When n A.Mean are inserted between a, b then the common difference of sequence $(d) = \frac{b-a}{n+1}$

⊛ If (a, b, c) form an A.S then:

① $(a \pm k, b \pm k, c \pm k)$ form an A.S Too

② (ak, bk, ck) form an A.S Too

③ if $(a_1, b_1, c_1, ---)$ A.S common d_1
$(a_2, b_2, c_2, ---)$ A.S common d_2
Then $(a_1 + a_2, b_1 + b_2, c_1 + c_2 ---)$ is A.S common $d_1 + d_2$

Geometric sequence:

$r \neq 0 \rightarrow$ Common ratio $\frac{T_{n+1}}{T_n}$

increasing: $r > 1$, $a > 0$ or $0 < r < 1$, $a < 0$

decreasing $0 < r < 1$, $a > 0$, or $r > 1$, $a < 0$

Alternating sign if $r < 0$

Constant $r = 1$

⊛ $T_n = a r^{n-1}$ ⊛ $L = a r^{n-1}$ ⊛ if x, y, z G.S $\therefore y^2 = xz$

⊛ $S_n = \frac{a(1-r^n)}{1-r}$, $r \neq 1$ or $\frac{a(r^n-1)}{r-1}$

⊛ $S_n = \frac{a - Lr}{1-r}$ or $\frac{Lr - a}{r-1}$ ⊛ $S_\infty = \frac{a}{1-r}$

$-1 < r < 1$ مهم جداً

⊛ if T_x, T_y are two terms of G.S Then $r^{x-y} = \frac{T_x}{T_y}$

⊛ The Arithmetic mean between two different positive real numbers is greater Than Their geometric mean.

$\frac{x+y}{2} > \sqrt{xy}$

⊛ IF $(a, b, c, \ldots)$ is G.S Common (r)

$(x, y, z, \ldots)$ is G.S Common (m) Then

(1) $(ax, by, cz, \ldots)$ is G.S Common (rm)

(2) $(ak, bk, ck, \ldots)$ is G.S Common r, $k \neq 0$

(3) $(\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \ldots)$ is G.S Common r

(4) $(a^k, b^k, c^k, \ldots)$ is G.S Common r^k

$n, r \in N$, $r \leq n$ Then

$\Rightarrow \underline{|n} = n(n-1)(n-2) \ldots \times 3 \times 2 \times 1$ or $\underline{|n} = n \underline{|n-1}$

$\Rightarrow {}^nP_r = n(n-1)(n-2) \ldots (n-r+1) = \frac{\underline{|n}}{\underline{|n-r}} \in Z^+$ $1 \leq r \leq n$

$\Rightarrow {}^nP_n = {}^nP_{n-1} = \underline{|n}$

$\Rightarrow {}^nC_r = \frac{{}^nP_r}{\underline{|r}} = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$ $\Rightarrow {}^nC_r = {}^nC_{n-r}$ $\Rightarrow {}^nC_0 = {}^nC_n = 1$

$\Rightarrow {}^nC_1 = n$ $\Rightarrow$ IF ${}^nC_x = {}^nC_y$ Then $x = y$ or $x + y = n$

كلمة and $\times$ ، or $+$ ، ترتيب مهم يبقى P ، ترتيب مش مهم C

Second: Calculus

① $V(h) = F(x+h) - F(x)$ ② $A(h) = \frac{V(h)}{h} = \frac{F(x+h) - F(x)}{h}$

③ rate of change $= \lim_{h \to 0} A(h) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$

$= \text{slope} = m = \tan\theta = y' = F'(x) = \frac{dy}{dx} = \frac{\Delta y}{\Delta x}$

$y = mx + c$; with + direction x-axis

$= \frac{y_2 - y_1}{x_2 - x_1} = -\frac{\text{coff } x}{\text{coff } y} = -\frac{b}{a}$

Two points ; $ax + by = c$; $\vec{u}(a, b)$ direction

④ $L_1 \parallel L_2 \quad \therefore m_1 = m_2$ ⑤ $L_1 \perp L_2 \quad \therefore m_1 \times m_2 = -1$

⑥ $L \parallel x$-axis slope $= 0$, $L \parallel y$-axis slope undefined

⑦ Cut with x-axis but $y = 0$

Cut with y-axis but $x = 0$

⑧ equation of tangent $\frac{y - y_1}{x - x_1} = \text{slope}$

equation of the normal $\frac{y - y_1}{x - x_1} = \frac{-1}{\text{slope}}$

في حاله drivative ينزل Power وتقل 1

$\Rightarrow F(x) = (\quad)^n$

$F'(x) = n(\quad)^{n-1}$ في drivative اللي جوه

$\Rightarrow F(x) = \sqrt{\quad}$ $\therefore F'(x) = \frac{\text{drivative اللي جوه}}{2\sqrt{\quad}}$

$\Rightarrow F(a)^+ = F(a)^- \quad \therefore$ exist

$F(a)^+ = F(a)^- = F(a)$ Continuous

$F'(a)^+ = F'(a)^-$ is diff.

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$\Rightarrow y = f \circ g(x) = f(g(x))$ Then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$.

$\Rightarrow y = \frac{f(x)}{g(x)}$, $g(x) \neq 0$, then:

$$\frac{dy}{dx} = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

$\Rightarrow$ Chain rule: $y = f(z)$, $z = g(x)$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$\Rightarrow \frac{d}{dx}(y^n) = n y^{n-1} \left(\frac{dy}{dx}\right)$

• Derivatives of trigonometric functions:

• $y = \sin x$, then $\frac{dy}{dx} = \cos x$ • derivativ angle

• $y = \cos$, " $y' = -\sin x$ • " "

• $y = \tan x$, " $y' = \sec^2 x$ • " "

• Remember that: $\sin^2 x + \cos^2 x = 1$

• Slope "m" $\longrightarrow$ "first derivativ"

① St: $ax + by + c$

$m = \frac{-\text{coeff } x}{\text{coeff } y} = \frac{-a}{b}$

② two Points $(x_1, y_1), (x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

③ angle θ

$m = \tan\theta$

④ $\vec{u} = (a, b)$

"$\vec{u}$ direction vector"

$\therefore m = \frac{b}{a}$

• Slope of x-axis = zero

• Slope of y-axis = $\frac{1}{0}$

* $L_1 \parallel L_2 \longrightarrow m_1 = m_2$

* $L_1 \perp L_2 \longrightarrow m_1 \times m_2 = -1$

• equation of St:

① Given a Point onit (x_1, y_1) and Slope (m) is: $y - y_1 = m(x - x_1)$

② Given the Slope (m) and the intercepted from y-axis is:
$y = mx + c$

③ Given the two intercepted Parts from the two coordinate is: $\frac{x}{a} + \frac{y}{b} = 1$

④ equation of st line // to x-axis and passes through the point (L, K) is $\boxed{y = K}$

⑤ equation of st line // to y-axis and passes through the point (L, K) is $\boxed{x = L}$

⑥ The equation of x-axis is $\boxed{y = 0}$

⑦ The equation of y-axis is $\boxed{x = 0}$

⑧ " " " st line passes through origin point is: $\boxed{y = mx}$

- To find the points of intersection with the x-axis, put $\boxed{y = 0}$
- " " " " " " " y-axis, put $\boxed{x = 0}$

• integrations

(1) $\int K \, dx = Kx + c$ "Const"

(2) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

(3) $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

(4) $\int a f(x) \, dx = a \int f(x) \, dx$ "where a is a constant $\neq 0$"

(5) $\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

(6) $\int (f(x))^n \cdot f'(x) \, dx = \frac{(f(x))^{n+1}}{n+1} + c$

(7) $\int \sin(ax+b) \, dx = -\frac{1}{a} \cos(ax+b) + c$

(8) $\int \cos(ax+b) \, dx = \frac{1}{a} \sin(ax+b) + c$

(9) $\int \sec^2(ax+b) \, dx = \frac{1}{a} \tan(ax+b) + c$

• Remember:

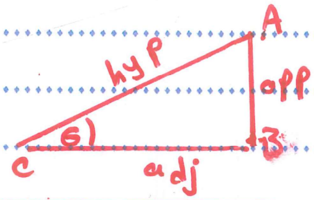
- $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$
- $\sin^2 x + \cos^2 x = 1$, $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = 1 - 2\sin^2 x$
  $= 2\cos^2 x - 1$
- $\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$, • $\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ • $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

• Remember:

$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$

$\tan\theta = \dfrac{opp}{adj}$



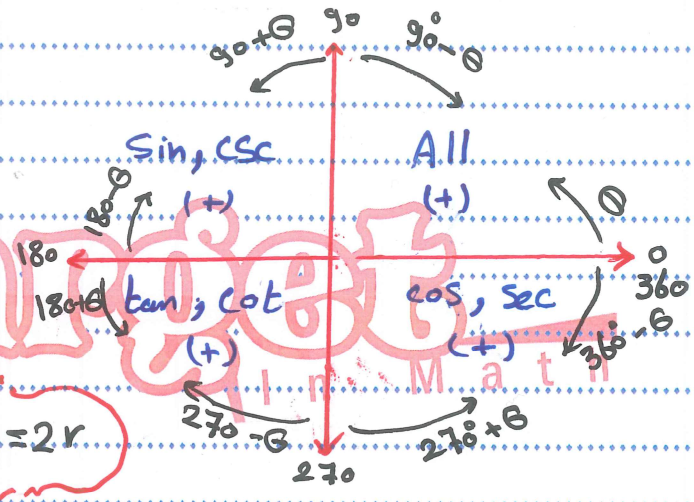
أي حاجة X مقلوبها = 1

• $\sin\theta \csc\theta = 1$, $\cos\theta \sec\theta = 1$

$\tan\theta \cot\theta = 1$

(90, 270) → change

(180, 360) → not change



• Sine rule:

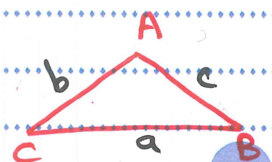
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

• Cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = c^2 + a^2 - 2ca\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

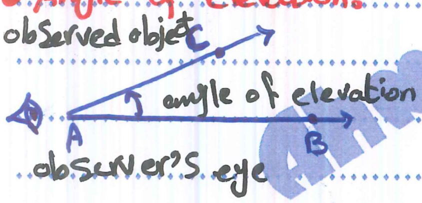


$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

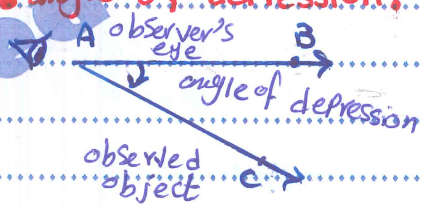
$\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

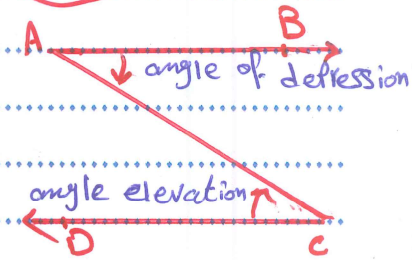
$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

• Angle of elevation:



• angle of depression:





• Trigonometric functions of double of measure of an angle:

• $\sin 2A = 2\sin A\cos A$

• $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

• $\cos 2A = \cos^2 A - \sin^2 A$

$2\cos^2 A - 1$

$1 - 2\sin^2 A$

• Heron's formula to find area of △:

$2b = a + b + c$

$a = \sqrt{P(P-a)(P-b)(P-C)}$

(radius) $\leftarrow r = \dfrac{\triangle}{P}$